Applications of the Poisson distribution

Ali Omar

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1. Introduction

The distributions of random variables have attracted a lot of interest in recent years. Their probability density functions, in a real variable x and in a complex variable z, have played an important role in statistics and probability theory. For this reason, the distributions have been extensively studied. Many types of distributions have emerged from real life situations such as binomial distribution, Poisson distribution, geometric distribution, hypergeometric distribution, and negative binomial distribution.

A random variable x follows a Poisson distribution if its probability density function (PDF) is given by:

$$f(x) = \frac{e^{-m}}{x!} m^x, x = 0,1,2, \dots \qquad (1.1)$$

For the parameter of the distribution *m*. The Poisson distribution started receiving interest in the theory of univalent functions, firstly by Porwal [8] and then later by Porwal and Dixit [9] who provided moments and moments' generating functions with the Mittag-Leffler Poisson distribution.

We denote by $\mathcal{A}$ the well-known class of the normalized functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \qquad (1,2)$$

Functions that are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C}: |z| < 1\}$.

We also let $\mathcal{T}$ be a subclass of $\mathcal{A}$ consisting of functions of the form,

$$(z) = z - \sum_{n=2}^{\infty} |a_n| z^n,\ z \in \mathbb{U}. \qquad (1.3)$$

Now, we recall the definitions of the classes $k - \mathcal{ST}[A, B]$ and $k - \mathcal{UCV}[A, B]$ that were introduced and studied by Noor and Malik [4].

A function $f \in \mathcal{A}$ is said to be in the class k-Janowski starlike functions, denoted by $k - \mathcal{ST}[A, B], k \geq 0, -1 \leq B < A \leq 1$, if and only if

$$\Re\left(\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)}\right) > k\left|\frac{(B-1)\frac{zf'(z)}{f(z)} - (A-1)}{(B+1)\frac{zf'(z)}{f(z)} - (A+1)} - 1\right|. \qquad (1.4)$$

Further, a function $f \in \mathcal{A}$ is said to be in the class k-Janowski convex functions $k\ \mathcal{UCV}[A,B], k \geq 0, -1 \leq B < A \leq 1$, if and only if

$$\Re\left(\frac{(B-1)\frac{(zf'(z))'}{f'(z)}-(A-1)}{(B+1)\frac{(zf'(z))'}{f'(z)}-(A+1)}\right) > k\left|\frac{(B-1)\frac{(zf'(z))'}{f'(z)}-(A-1)}{(B+1)\frac{(zf'(z))'}{f'(z)}-(A+1)}-1\right|, \tag{1.5}$$

clearly

$$f(z) \in k-\mathcal{UCV}[A,B] \Leftrightarrow zf'(z) \in k-\mathcal{ST}[A,B].$$

The above are generalizations of the following special cases:

(1) $k-\mathcal{ST}[1,-1]=k-\mathcal{ST}$ and $k-\mathcal{UCV}[1,-1]=k-\mathcal{UCV}$, the well-known classes of k starlike and k-uniformly convex functions respectively, introduced by Kanas and Wisniowska [6,7 and also 1]

(2) $k-\mathcal{ST}[1-2\gamma,-1]=k-\mathcal{SD}[k,\gamma]$ and $k-\mathcal{UCV}[1-2\gamma,-1]=k-\mathcal{KD}[k,\gamma]$, the classes introduced by Shams et al. in [10].

(3) $0-\mathcal{ST}[A,B]=\mathrm{S}^*[A,B]$ and $0-\mathcal{UCV}[A,B]=\mathcal{C}[A,B]$ the well-known classes of Janowski starlike and Janowski convex functions respectively, introduced by Janowski [12]

(4) $0-\mathcal{ST}[1-2\gamma,-1]=\mathcal{S}^*(\gamma)$ and $0-\mathcal{UCV}[1-2\gamma,-1]=\mathcal{C}(\gamma)$, the well-known classes of starlike functions of order $\gamma(0 \leq \gamma < 1)$ and convex functions of order $\gamma(0 \leq \gamma < 1)$ respectively, (see [3]).

If $f(z) \in k-\mathcal{ST}[A,B]$ then

$$w=\frac{(B-1)\frac{zf'(z)}{f(z)}-(A-1)}{(B+1)\frac{zf'(z)}{f(z)}-(A+1)}$$

takes all values from the domain $\Omega_k, k \geq 0$ as

$$\begin{aligned}\Omega_k &= \{w: \Re w > k|w-1|\}\\ &= \left\{u+iv: u > k\sqrt{(u-1)^2+v^2}\right\}\end{aligned}$$

The domain Ω_k represents the right half plane for $k=0$; a hyperbola for $0<k<1$; a parabola for $k=1$ and an ellipse for $k>1$, (see [4]).

A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{R}^\tau(C,D), \tau \in \mathbb{C}\setminus\{0\}, -1 \leq D < C \leq 1$, if it satisfies the inequality

$$\left|\frac{f'(z)-1}{(C-D)\tau-D[f'(z)-1]}\right| < 1,\ z \in \mathbb{U}$$

The class above was introduced by Dixit and Pal [13] providing the following results

Lemma 1.1. *[13] If $f \in \mathcal{R}^{\tau}(C,D)$ is of the form (1.2), then*

$$|a_n| \le (C-D)\frac{|\tau|}{n},\ n \in \mathbb{N} \setminus \{1\}$$

The result is sharp for the function

$$f(z) = \int_0^z \left(1 + \frac{(C-D)|\tau|t^{n-1}}{1+Dt^{n-1}}\right)dt,\ (z \in \mathbb{U}; n \in \mathbb{N} \setminus \{1\}).$$

The well-known Mittag-Leffler function $E_\alpha(z)$ studied by Mittag-Leffler [2] and given by

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)},\ (z \in \mathbb{C}, \Re(\alpha) > 0).$$

Prabhakar [5, 11] has generalized the Mittag – Leffler function as follows

$$E_{\alpha,\beta}^{\theta}(z) := \sum_{n=0}^{\infty} \frac{(\theta)_n}{\Gamma(\alpha n + \beta)}.\ \frac{z^n}{n!},\quad z, \beta, \theta \in \mathbb{C}\ ;\ \Re(\alpha) > 0,$$

here $(\theta)_v$ denotes the familiar Pochhammer symbol defined as

$$(\theta)_v \coloneqq \frac{\Gamma(\theta + v)}{\Gamma(\theta)} = \begin{cases} 1, & if\ v = 0,\ \ \theta \in \mathbb{C}\backslash\{0\} \\ \theta(\theta+1)\dots(\theta+n-1), & if\ v = n \in N, \theta \in \mathbb{C} \end{cases}$$

$$(1)_n = n!,\qquad n \in N_0, N_0 = N \cup \{0\},\qquad N = \{1,2,3,\dots\}.$$

Since the generalized that Mittag-Leffler function $E_{\alpha,\beta}^{\theta}(z)$ does not belong to the family $\mathcal{A}$. Let us consider the following normalization of the Mittag-Leffler function

$$\mathbb{E}_{\alpha,\beta}^{\theta}(z) = \Gamma(\beta) z E_{\alpha,\beta}^{\theta}(z)$$

$$= z + \sum_{n=2}^{\infty} \frac{(\theta)_n \Gamma(\beta)}{n!\,\Gamma(\alpha(n-1)+\beta)} z^n \tag{1.6}$$

where $z, \alpha, \beta \in \mathbb{C}; \beta \neq 0, -1, -2, \cdots$ and $\Re(\beta) > 0, \Re(\alpha) > 0$.

Our attention in this paper is only to the cases where α, β are real-valued and $z \in \mathbb{U}$.

The probability mass function of the generalized Mittag-Leffler-type Poisson distribution will be then given by

$$P(x = r) = \frac{m^r}{\Gamma(\alpha k + \beta)\mathbb{E}_{\alpha,\beta}^{\theta}(m)},\ r = 0,1,2,3,\cdots,$$

where $m > 0, \alpha > 0$ and $\beta > 0$. Using the normalized form of Mittag-Leffler function in (1.6), the one can introduce a power series whose coefficients are probabilities of the generalized Mittag-Leffler-type Poisson distribution series, as:

$$H_{\alpha,\beta}^{m,\theta}(z) := z + \sum_{n=2}^{\infty} \frac{(\theta)_n \Gamma(\beta) m^{n-1}}{n!\, \Gamma(\alpha(n-1)+\beta)\mathbb{E}_{\alpha,\beta}^{\theta}(m)} z^n,\ z \in \mathbb{U}$$

To serve our purpose, we also need to define the series

$$I_{\alpha,\beta}^{m,\theta}(z) := 2z - H_{\alpha,\beta}^{m}(z) = z - \sum_{n=2}^{\infty} \frac{(\theta)_n \Gamma(\beta) m^{n-1}}{\mathrm{n}!\, \Gamma(\alpha(n-1)+\beta)\mathbb{E}_{\alpha,\beta}^{\theta}(m)} z^n,\ z \in \mathbb{U} \tag{1.7}$$

Finally, and by the means of the convolution we deduce the following operator:

$$\mathcal{J}_{\alpha,\beta}^{m,\theta} f(z) = H_{\alpha,\beta}^{m,\theta}(z) * f(z) = z + \sum_{n=2}^{\infty} \frac{(\theta)_n \Gamma(\beta) m^{n-1}}{\mathrm{n}!\, \Gamma(\alpha(n-1)+\beta)\mathbb{E}_{\alpha,\beta}^{\theta}(m)} a_n z^n,\ z \in \mathbb{U},$$

2. Inclusion Results of $I_{\alpha,\beta}^{m,\theta}(z)$

To establish our main results, we shall require the following lemmas.

Lemma 2.1. *[4] A function f of the form (1.2) is in the class $k - \mathcal{ST}[A,B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} [2(k+1)(n-1) + |n(B+1) - (A+1)|]|a_n| \le |B - A| \tag{2.1}$$

where $-1 \le B < A \le 1$ and $k \ge 0$.

Lemma 2.2. *[4] A function f of the form (1.2) is in the class $k - \mathcal{UCV}[A,B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} n[2(k+1)(n-1) + |n(B+1) - (A+1)|]|a_n| \le |B - A| \tag{2.2}$$

where $-1 \le B < A \le 1$ and $k \ge 0$.

Unless otherwise mentioned, we shall assume in this paper that $\alpha, m > 0, k \ge 0$ and $-1 \le B < A \le 1$.

Theorem 2.3. *Let $\beta > 1$. Then $I_{\alpha,\beta}^{m,\theta} \in k - \mathcal{ST}[A,B]$ if*

$$\frac{(\theta)_n\Gamma(\beta)}{n!\,\mathbb{E}^{\theta}_{\alpha,\beta}(m)}\Big[\frac{2k+B+3}{\alpha}\Big(E_{\alpha,\beta-1}(m)-\frac{1}{\Gamma(\beta-1)}\Big)$$
$$+\Big[\Big(\frac{2k+B+3}{\alpha}\Big)(1-\beta)+(B+A+2)\Big]\Big(E^{\theta}_{\alpha,\beta}(m)-\frac{n!}{(\theta)_n\Gamma(\beta)}\Big)\Big]$$
$$\leq \quad |B-A| \tag{2.3}$$

Proof. In view of Lemma 2.1 *and (2.1) it suffices to show that*

$$J_1 := \sum_{n=2}^{\infty} [2(k+1)(n-1)+|n(B+1)-(A+1)|]\frac{(\theta)_n\Gamma(\beta)m^{n-1}}{\mathrm{n}!\,\Gamma(\alpha(n-1)+\beta)\mathbb{E}_{\alpha,\beta}(m)} \leq |B-A|$$

We have

$$J_1 \leq \sum_{n=2}^{\infty} [2(k+1)(n-1)+n(B+1)+(A+1)]\frac{(\theta)_n\Gamma(\beta)m^{n-1}}{\mathrm{n}!\,\Gamma(\alpha(n-1)+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}$$

$$=\sum_{n=2}^{\infty} [(2k+B+3)n+(A-2k-1)]\frac{(\theta)_n\Gamma(\beta)m^{n-1}}{\mathrm{n}!\,\Gamma(\alpha(n-1)+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}$$

$$=\sum_{n=1}^{\infty} [(2k+B+3)(n+1)+(A-2k-1)]\frac{(\theta)_n\Gamma(\beta)m^{n}}{\mathrm{n}!\,\Gamma(\alpha n+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}$$

$$=\sum_{n=1}^{\infty} [(2k+B+3)n+(B+A+2)]\frac{(\theta)_n\Gamma(\beta)m^{n}}{\mathrm{n}!\,\Gamma(\alpha n+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}$$

$$=\Big(\frac{2k+B+3}{\alpha}\Big)\sum_{n=1}^{\infty} [(\alpha n+\beta-1)+(1-\beta)]\frac{(\theta)_n\Gamma(\beta)m^{n}}{\mathrm{n}!\,\Gamma(\alpha n+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}$$

$$+(B+A+2)\sum_{n=1}^{\infty} \frac{(\theta)_n\Gamma(\beta)m^{n}}{\mathrm{n}!\,\Gamma(\alpha n+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}$$

$$=\Big(\frac{2k+B+3}{\alpha}\Big)\sum_{n=1}^{\infty} \frac{(\theta)_n\Gamma(\beta)m^{n}}{\mathrm{n}!\,\Gamma(\alpha n+\beta-1)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}$$

$$+\Big[\Big(\frac{2k+B+3}{\alpha}\Big)(1-\beta)+(B+A+2)\Big]\sum_{n=1}^{\infty} \frac{(\theta)_n\Gamma(\beta)m^{n}}{\mathrm{n}!\,\Gamma(\alpha n+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}$$

$$=\frac{(\theta)_n\Gamma(\beta)}{n!\,\mathbb{E}^{\theta}_{\alpha,\beta}(m)}\Big[\frac{2k+B+3}{\alpha}\Big(E^{\theta}_{\alpha,\beta-1}(m)-\frac{1}{\Gamma(\beta-1)}\Big)$$

$$+\left[\left(\frac{2k+B+3}{\alpha}\right)(1-\beta)+(B+A+2)\right]\left(E^{\theta}_{\alpha,\beta}(m)-\frac{n!}{(\theta)_n\Gamma(\beta)}\right)\Bigg]$$

$$\leq |B-A|,$$

This complete the proof of Theorem 2.3. □

Theorem 2.4. *Let* $\beta > 2$. *Then* $I^{m,\theta}_{\alpha,\beta} \in k-\mathcal{UCV}[A,B]$ *if*

$$\frac{(\theta)_n\Gamma(\beta)}{n!\,\mathbb{E}^{\theta}_{\alpha,\beta}(m)}\Bigg[\frac{2k+B+3}{\alpha^2}\left(E^{\theta}_{\alpha,\beta-2}(m)-\frac{1}{\Gamma(\beta-2)}\right)$$
$$+\left(\frac{(2k+B+3)(3-2\beta)+\alpha(2B+A+2k+5)}{\alpha^2}\right)\left(E^{\theta}_{\alpha,\beta-1}(m)-\frac{1}{\Gamma(\beta-1)}\right)$$
$$+\left(\frac{(2k+B+3)(1-\beta)^2}{\alpha^2}+\frac{(2B+A+2k+5)(1-\beta)}{\alpha}+(B+A+2)\right)\left(E^{\theta}_{\alpha,\beta}(m)-\frac{n!}{(\theta)_n\Gamma(\beta)}\right)\Bigg]$$
$$\leq |B-A|$$

Proof. We consider the same approach of Theorem 2.3 by the means of Lemma 2.2 and (2.2). Here we let

$$J_2:=\sum_{n=2}^{\infty} n[2(k+1)(n-1)+|n(B+1)-(A+1)|]\frac{(\theta)_n\Gamma(\beta)m^{n-1}}{\mathrm{n}!\,\Gamma(\alpha(n-1)+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)} \leq |B-A|.$$

3. Inclusion Results of $\mathcal{I}^{m}_{\alpha,\beta}f$

Theorem 3.1. *Let* $\beta > 1$. *If* $f \in \mathcal{R}^{\tau}(C,D)$, *then* $\mathcal{I}^{m,\theta}_{\alpha,\beta}f \in k-\mathcal{UCV}[A,B]$ *if*

$$\frac{(C-D)|\tau|(\theta)_n\Gamma(\beta)}{n!\,\mathbb{E}^{\theta}_{\alpha,\beta}(m)}\Bigg[\frac{2k+B+3}{\alpha}\left(E_{\alpha,\beta-1}(m)-\frac{1}{\Gamma(\beta-1)}\right)$$
$$+\left[\left(\frac{2k+B+3}{\alpha}\right)(1-\beta)+(B+A+2)\right]\left(E_{\alpha,\beta}(m)-\frac{n!}{(\theta)_n\Gamma(\beta)}\right)\Bigg]$$
$$\leq \quad |B-A| \tag{1.3}$$

Proof. Using Lemma 2.2 and (2.1) it is enough to verify that

$$\sum_{n=2}^{\infty} n[2(k+1)(n-1)+|n(B+1)-(A+1)|]\frac{(\theta)_n\Gamma(\beta)m^{n-1}}{\mathrm{n}!\,\Gamma(\alpha(n-1)+\beta)\mathbb{E}^{\theta}_{\alpha,\beta}(m)}|a_n| \leq |B-A|$$

Now, since $f \in \mathcal{R}^{\tau}(C,D)$, in view of Lemma 1.1 the coefficients bound is

$$|a_n| \leq \frac{(C-D)|\tau|}{n}, n \in \mathbb{N} \setminus \{1\}$$

Thus, it is sufficient to show that

$$(C-D)|\tau|\left[\sum_{n=2}^{\infty} [2(k+1)(n-1)+|n(B+1)-(A+1)|]\frac{(\theta)_n\Gamma(\beta)m^{n-1}}{n!\,\Gamma(\alpha(n-1)+\beta)\mathbb{E}_{\alpha,\beta}(m)}\right]$$
$$\leq \quad |B-A|.$$

Which is the same approach of the proof of Theorem 2.3 we conclude that $\mathcal{J}_{\alpha,\beta}^{m} f \in k-\mathcal{UCV}[A,B]$ if (3.1) holds true. □

4. Inclusion results of the integral operator $\mathcal{G}_{\alpha,\beta}^{m,\theta}$

Following the same previous methods, we can readily deduce the next result

Theorem 4.1. *If $\beta > 1$, then the integral operator*

$$\mathcal{G}_{\alpha,\beta}^{m,\theta}(z) := \int_0^z \frac{I_{\alpha,\beta}^{m,\theta}(t)}{t} dt, z \in \mathbb{U},$$

is in k-$\mathcal{UCV}[A,B]$ if the inequality (2.3) is satisfied.

Proof. By the assumption (1.7) we have

$$\mathcal{G}_{\alpha,\beta}^{m,\theta}(z) = z - \sum_{n=2}^{\infty} \frac{(\theta)_n\Gamma(\beta)m^{n-1}}{(\theta)_n\Gamma(\alpha(n-1)+\beta)\mathbb{E}_{\alpha,\beta}^{\theta}(m)}\frac{z^n}{n}.$$

Now, using Lemma 2.2 and (2.1), the integral operator $\mathcal{G}_{\alpha,\beta}^{m}(z)$ belongs to $k-\mathcal{UCV}[A,B]$ if

$$\sum_{n=2}^{\infty} [2(k+1)(n-1)+|n(B+1)-(A+1)|]\frac{(\theta)_n\Gamma(\beta)m^{n-1}}{n!\,\Gamma(\alpha(n-1)+\beta)\mathbb{E}_{\alpha,\beta}^{\theta}(m)} \leq |B-A|$$

we conclude that $\mathcal{G}_{\alpha,\beta}^{m,\theta} \in k-\mathcal{UCV}[A,B]$ if (2.3) holds true. □

5. Special cases

Let $A = 1-2\gamma$, and $B = -1$ with $0 \leq \gamma < 1$ in the above theorems, we receive the following special cases:

Corollary 5.1. *Let $\beta > 1$. Then $I_{\alpha,\beta}^{m,\theta} \in k-\mathcal{SD}[k,\gamma]$ if*

$$
\begin{aligned}
&\frac{(\theta)_n\Gamma(\beta)}{n!\,\mathbb{E}^{\theta}_{\alpha,\beta}(m)}\Bigg[\frac{k+1}{\alpha}\left(E^{\theta}_{\alpha,\beta-1}(m)-\frac{1}{\Gamma(\beta-1)}\right)\\
&+\left[\left(\frac{k+1}{\alpha}\right)(1-\beta)+1-\gamma\right]\left(E^{\theta}_{\alpha,\beta}(m)-\frac{n!}{(\theta)_n\Gamma(\beta)}\right)\Bigg]\\
\le\quad &1-\gamma.
\end{aligned}
$$

Corollary 5.2. *Let* $\beta>2$. *Then* $I^{m,\theta}_{\alpha,\beta}\in k-\mathcal{KD}[k,\gamma]$ *if*

$$
\begin{aligned}
&\frac{(\theta)_n\Gamma(\beta)}{n!\,\mathbb{E}_{\alpha,\beta}(m)}\Bigg[\frac{k+1}{\alpha^2}\left(E^{\theta}_{\alpha,\beta-2}(m)-\frac{1}{\Gamma(\beta-2)}\right)\\
&+\left(\frac{(k+1)(3-2\beta)+\alpha(2-\gamma+k)}{\alpha^2}\right)\left(E^{\theta}_{\alpha,\beta-1}(m)-\frac{1}{\Gamma(\beta-1)}\right)\\
&+\left(\frac{(k+1)(1-\beta)^2}{\alpha^2}+\frac{(2-\gamma+k)(1-\beta)}{\alpha}+(1-\alpha)\right)\left(E^{\theta}_{\alpha,\beta}(m)-\frac{n!}{(\theta)_n\Gamma(\beta)}\right)\Bigg]\\
&1-\gamma
\end{aligned}
$$

Corollary 5.3. *Let* $\beta>1$. *If* $f\in\mathcal{R}^{\tau}(C,D)$, *then* $\mathcal{J}^{m,\theta}_{\alpha,\beta}f\in k-\mathcal{KD}[k,\gamma]$ *if*

$$
\begin{aligned}
&\frac{(C-D)|\tau|(\theta)_n\Gamma(\beta)}{n!\,\mathbb{E}_{\alpha,\beta}(m)}\Bigg[\frac{k+1}{\alpha}\left(E^{\theta}_{\alpha,\beta-1}(m)-\frac{1}{\Gamma(\beta-1)}\right)\\
&+\left[\left(\frac{k+1}{\alpha}\right)(1-\beta)+1-\gamma\right]\left(E^{\theta}_{\alpha,\beta}(m)-\frac{n!}{(\theta)_n\Gamma(\beta)}\right)\Bigg]\\
\le\quad &1-\gamma
\end{aligned}
$$

Corollary 5.4. *Let* $\beta>1$. *Then the integral operator given by (4.1) is in the class* $k\ \mathcal{KD}[k,\gamma]$ *if the inequality in Corollary 5.1 holds true.*

6. Conclusion

The generalized Mittag-Leffler function has been investigated by the means of Poisson distribution. A normalized form $\mathbb{E}^{\theta}_{\alpha,\beta}(z)$ has been studied in terms of its inclusion in the well know subclasses of analytic functions, here we have considered $k-\mathcal{ST}[A,B]$ and $k-\mathcal{UCV}[A,B]$. Sufficient conditions are derived for $\mathrm{I}^{m,\theta}_{\alpha,\beta}(z)$, $\mathcal{J}^{m}_{\alpha,\beta}f$ and the integral operator $\mathcal{G}^{m,\theta}_{\alpha,\beta}$ to belong to k-uniformly Janowski starlike and k-Janowski convex functions. Finally, for some values of the parameters A and B special cases are discussed.

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